Behaviour of Gases

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1 Outlines of the measurement

——————————- $1Watt = 683lumens$ for light at $555nm$ ——————————

X scientists from Earth were commissioned to understand the behaviour of gases in a gas giant that is very close to a super-hot star. However may it be impossible for the crew to land on a planet of gas, they will be conducting their experiments in orbit. The scientists are well protected from any sort of hazardous exposure. The first scientist has to prepare the suitable measuring devices for the experiments. Since the team is dealing with systems in extreme conditions, they agreed to measure heat with the assistance of a matter that emits black-body radiation. For this reason, we will be using the equation for the energy intensity per the wavelength, that follows;

$$E(\lambda, T) = \frac{2hc^2}{\lambda^5} \cdot \frac{1}{e^{\frac{hc}{\lambda k_b T}} - 1} \tag{1}$$

Called the Planck equation, it is a function of temperature and the wavelength of the radiated light. The following contraption is built in hopes of determining various properties.

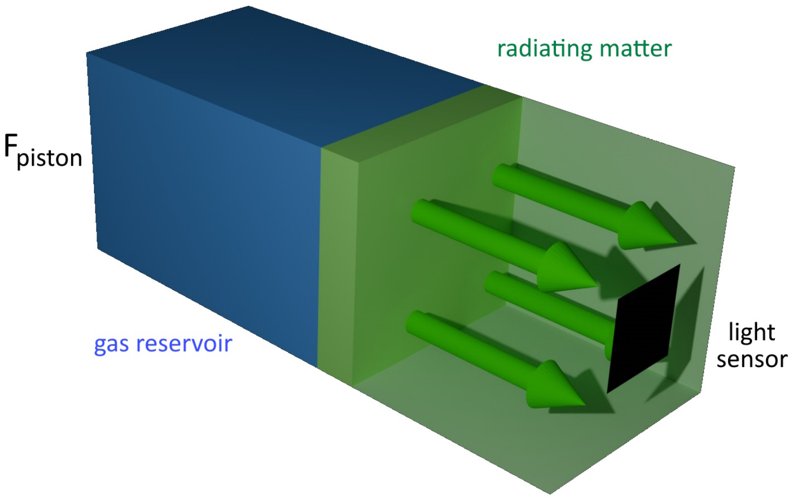

Figure 1: The experiment contraption

1.i Find the units of E.

1.ii Given that this matter radiates E_0 at 500°K, find the ratio of E between initial conditions and after the temperature is tripled.
Hint: assume symmetry arguments for the visualised experiment contraption.

1.iii Calculate how hot the matter has to be to radiate in the same intensity at 500°K, 555 nm; for the color red (660 nm).

Consider an LED sign board that consists of 555 nm LED's where 5% of the diodes malfunction. This board consists of 200 parallel units of 120 diodes connected in series, which are 15.5 mm apart from each other. Each LED emits 586.6 lumen per watt in the expense of 20mA of forward current with 5mV voltage drop.

1.iv Calculate the minimum voltage that the energy source has to provide to fully light the board.

1.v Calculate the ratio between the black-body radiation intensity for 555 nm at 500°K and the LED board; Assume planar radiation is present even in the presence of malfunctioned diodes.

1.vi For later use, estimate the heat capacities at constant volume of two arbitrary tri-atomic gases that have $D_{\infty h}$ and C_{2v} symmetry, respectively. It is known that each vibrational mode contributes R to the heat capacity where there are 4 and 3 vibrational modes for the aforementioned gases, respectively.

1.vii

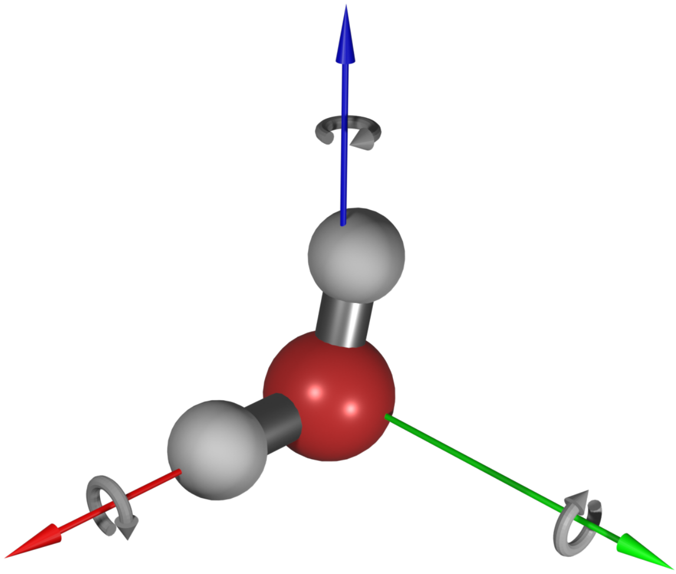

Figure 2: Translational and rotational degrees of freedom for a triatomic molecule of C_{2v} symmetry

2 Onto thermodynamics

The second scientist has to to the thermodynamic calculations regarding some gas. In this gas giant, however; they observed that the ideal gas law does not hold. Instead, upon basic examination, the second scientist were able to find 3 relations that is enough to derive the ideal gas law for this planet. It follows that:

$$T \propto V^2 \text{ at constant } P,\, n \tag{2}$$
$$n \propto V^2 \text{ at constant } P,\, T \tag{3}$$
$$P \propto \frac{1}{V^2} \text{ at constant } n,\, T \tag{4}$$

2.i Derive the ideal gas law for this planet, denote the gas constant as Y.

For the rest of this question, denote $Y = R\epsilon$, where R is the standard gas constant and ϵ is a constant property that is related to the specificity of the gas itself.

2.ii Decipher the units of Y and ϵ.

Now as we have derived the ideal gas law, it is time to do some thermodynamics. Further research shows that the definition of internal energy, work and heat are the same here as in Earth.

2.iii Formulate the isothermal reversible expansion work for this planet.

1 mole of gas X was injected into the formally empty gas reservoir at 500°C. ($P_{external} = 1.2atm, \epsilon_X = 0.0316[?]$)

2.iv Find the volume of the gas reservoir in litres. Consider the piston is free to move.

For the second scientist to do any further experiments, they are required to define H. It is known that the equation $dH = dU + d(nRT)$ holds. However, the scientist is unsure whether to use the classical expression of $d(nRT)$ or the counterpart where they consider the gas law that is specific to the gas giant.

2.v Write down the two possible mathematical definitions of dH

2.vi Derive the P-V relation for adiabatic processes regarding the new gas law. The simplified form *is required* to fit the format f(P)g(V)=K where functions f and g are arbitrary and K is a constant.
Hint: the relation PV^{γ} is derived from the mathematical expression of dH and the regarding constraints.

2.vii Derive the new definitions of ΔH_{new} for isobaric, isothermal, isochoric and isenthalpic conditions. Derive the new definition of dH_{new} for adiabatic conditions.

2.viii Determine the correct expression of dH for the compression of: a mole of a triatomic molecule of C_{2v} symmetry at its boiling point (373.69°K) under isochoric conditions in light of the evident:

$C_{matter} = 21\frac{J \cdot m^2}{g \cdot K}; d_{thickness} = 0.4mm; \rho_{matter} = 9\frac{kg}{L}$; Released heat is only captured by the radiating matter. The matter becomes hotter while the substance is at constant temperature, meaning the system drives away from thermodynamic equilibrium. Initially, every component is at thermodynamic equilibrium. After the piston push, they measure the lumen at start and end as $\Phi_i = 1.065 \cdot 10^{-12}$ and $\Phi_f = 1.134 \cdot 10^{-12}$ at $\lambda = 555nm$, respectively. They observe that only a little amount of gas is condensed into liquid form. For small molecules that don't make hydrogen bonds, Trouton's rule does not hold. Instead, the value follows the lines of $\Delta H \approx 30\frac{kJ}{mol}$. $V = 3L$, $P_i = 200atm$, $P_f = 25atm$, $\epsilon = 5.5$[?]

$$\frac{P_i}{P_f} = \left(\frac{V_f}{V_i}\right)^{\frac{1}{3}} \cdot e^{\left(\frac{C_v}{R} \cdot \left(\frac{V_f}{V_i}\right)\right)} \tag{5}$$

2.ix Supposing the equation above holds, approximate V_f for an adiabatic compression of a triatomic gas that has C_v symmetry using the third degree Taylor expansion for the Lambert W function and its property where $V_i = 25L, P_i = 75atm, P_f = 137atm$. The high pressure is obtained with the help of a mechanical push on the piston. The equations are as follows:

$$T_2\left(W\left(x\right)\right) = 1 + \frac{(x-e)}{2e} - \frac{3(x-e)^2}{2^4 \cdot e^2} + \frac{19(x-e)^3}{2^5 \cdot e^3 \cdot 3!} \qquad \text{at x=e} \tag{6}$$

$$W(x \cdot e^x) = x \tag{7}$$

2.x Is the approximation for the Lambert W function a good approximation in this case? Justify briefly.

2.xi Considering error of 2% and further is of significant error, could the scientist get a palatable P_f value if they were to calculate it by the other three properties obtained from before?